On the Non-Existence of Fundamental Ontic Positions: A Number-Theoretic No-Go Theorem

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Abstract

We investigate a broad class of physical theories in which fundamentally localizable entities are endowed with ontic positions that admit an injective numerical representation. Such theories include any framework in which spatial position constitutes a fundamental physical attribute independently of measurement and can, in principle, be represented exactly by numerical coordinates.

Starting from a small set of physically motivated structural axioms, we formulate a representation-independent measurement framework that distinguishes ontic positions from their numerical encoding. We show that every exact position measurement necessarily yields a finite numerical representation, while the underlying ontic position space itself is not assumed to be discrete.

The measurement framework induces a canonical arithmetic representation of all admissible exact position measurements. Once this canonical representation has been established, the remainder of the proof proceeds entirely within elementary number theory and requires no further physical assumptions. Under the stated axioms, we derive a contradiction showing that no ontic position-based theory admitting such an injective numerical representation can exist.

The contradiction is independent of dynamical laws, quantum-mechanical postulates, relativistic structure, probabilistic assumptions, and the particular realization of the measurement process. It therefore establishes a general number-theoretic no-go theorem for ontic position-based theories satisfying the stated axioms.

Keywords: ontic position; numerical representation; no-go theorem; measurement theory; quantum foundations; foundations of physics

1 Introduction

Physical theories routinely describe spatial positions by assigning real numbers to physical systems. This numerical representation is generally regarded as self-evident and is therefore rarely examined as an independent structural assumption. In both classical and quantum physics, positional quantities are routinely modeled by elements of the real numbers, while the mathematical representation itself is generally assumed to be compatible with the existence of exact physical positions.

The present work investigates this assumption at a highly general level. Rather than analyzing a particular physical theory, we consider the entire class of physical theories in which fundamentally localizable entities possess ontic positions that admit an injective numerical representation. No assumptions are made regarding dynamics, trajectories, probabilistic laws, quantum mechanics, relativistic spacetime, or specific measurement models beyond the structural framework introduced below.

The present result differs fundamentally from established no-go theorems on particle localization, such as Malament's theorem, which rule out certain forms of relativistic localization under specific assumptions concerning relativistic covariance, energy, localizability, and locality [1].

An injective numerical representation assigns a unique numerical value to every ontic position and thereby provides an unambiguous numerical parameterization of the underlying position space. It is important to distinguish this numerical representation from the ontic positions themselves. The existence of physical positions is a statement about the physical world, whereas their numerical representation constitutes an additional mathematical structure introduced to describe that world. The central question of this work is whether these two structures can consistently coexist under a minimal set of physically motivated assumptions.

The question addressed in this work is therefore not whether physical systems can be localized operationally, but whether fundamentally localizable entities can consistently be assigned unique numerical position values under a minimal set of physically motivated structural assumptions.

To answer this question, we first introduce an axiomatic framework for ontic position-based theories. We then develop a representation-independent measurement framework that separates ontic positions from their numerical encoding. This framework establishes that exact position measurements necessarily yield finitely representable numerical results while making no assumption that the underlying ontic position space itself is discrete.

From these structural properties we derive a canonical arithmetic representation of exact position measurements. Once this representation has been established, the remainder of the proof proceeds entirely within elementary number theory. No additional physical assumptions enter the proof. The resulting contradiction therefore follows solely from the arithmetic structure implied by the preceding physical assumptions.

The obtained result establishes a general no-go theorem for ontic position-based theories satisfying the proposed axioms. Its consequences are independent of quantum mechanics, relativistic structure, dynamical laws, probabilistic assumptions, and specific measurement interactions.

Although the theorem is formulated for injectively representable ontic positions, many physical theories naturally formulate spatial position through coordinate-based numerical representations. Consequently, the class of ontologies escaping the theorem may be substantially more restricted than the formal statement alone suggests.

The contradiction obtained below is therefore not tied to any particular physical theory but follows solely from the coexistence of the stated structural assumptions.

2 Axioms, Hypothesis and Basic Definitions

Definition 2.1 (localizable entity)**.** A physical entity is called fundamentally localizable if, at every instant of time, it possesses a unique ontic spatial position that is independent of observation, measurement, or epistemic access. The existence of such a position constitutes an objective property of the physical system and does not depend on any measurement procedure or numerical representation.

Hypothesis 2.2 (ontic position)**.** The present work investigates physical theories satisfying the following ontic position hypothesis:

Every fundamentally localizable entity possesses a unique ontic position at every instant of time. This ontic position exists independently of measurement and admits an injective numerical representation.

The following axioms formalize this hypothesis.

Axiom 2.3 (Existence)**.** Every fundamentally localizable physical entity possesses an ontic position at every instant of time.

Axiom 2.4 (Uniqueness)**.** At every instant of time, each fundamentally localizable entity possesses exactly one ontic position.

Axiom 2.5 (Exact measurability)**.** In an idealized localization limit, where neither fundamental nor technical measurement limitations (e.g. uncertainty relations, detector resolution, or noise) are present, every ontic position is, in principle, exactly measurable.

This idealized limit serves solely as a structural reference model. No assumptions are made concerning the practical realization of such measurements.

Axiom 2.6 (Injective representation)**.** There exists a set X of ontic positions and an injective numerical representation

$$n : X \to \mathbb{R},$$

assigning a unique real number to every ontic position.

Injectivity ensures that distinct ontic positions always possess distinct numerical representations.

No assumptions are made regarding dynamics, probability, trajectories, or temporal evolution.

Unless explicitly stated otherwise, the analysis is restricted to one spatial dimension. The generalization to $\mathbb{R}^k$ is immediate since the proof is component-wise and does not rely on dimension-specific properties.

Definition 2.7 (Ontic Position System)**.** A physical system is called an ontic position system if it satisfies Axioms 2.3–2.6.

Accordingly, every fundamentally localizable entity possesses a unique ontic position at every instant of time. In the idealized localization limit introduced in Axiom 2.5, this position is, in principle, exactly measurable.

Definition 2.8 (Ontic Position-Based Theory)**.** A physical theory is called an ontic position-based theory if there exist

$$P : S \to X$$

and

$$n : X \to \mathbb{R},$$

where

- S denotes the state space,
- X denotes the ontic position space,
- P assigns an ontic position to every physical state,
- n is an injective numerical representation assigning a unique real number to every ontic position.

The mapping

$$P : S \to X$$

assigns ontic positions to physical states.

The mapping

$$n : X \to \mathbb{R}$$

assigns unique numerical representations to ontic positions.

This distinction between physical positions and their numerical representation is fundamental throughout the remainder of this work.

No assumptions are made concerning dynamics, trajectories, probabilistic structure, or temporal evolution.

3 Representation Framework for Exact Position Measurements

Having established the structural framework of ontic position-based theories, we now turn to the numerical representation of ontic positions in the context of physical measurements.

While the preceding axioms postulate the existence of ontic positions as objective properties of physical systems, measurements do not provide direct access to these ontic positions themselves. Instead, every measurement produces a numerical representation associated with a positional attribute.

The purpose of this section is therefore not to introduce additional physical assumptions but to formalize the numerical structure of exact position measurements. In particular, we distinguish between ontic positions, primary measurement results, and their subsequent mathematical interpretation. This distinction allows the physical assumptions introduced in the previous section to be translated into a precise arithmetic framework.

We shall show that every exact positional measurement necessarily admits a finite numerical representation. Importantly, this restriction concerns only the numerical description produced by the measurement process. It neither implies any discreteness of the underlying ontic position space nor does it introduce any additional assumption concerning ontic localization beyond the axioms already introduced.

Once this representation framework has been established, all subsequent arguments leading to the no-go theorem become purely number-theoretic.

Definition 3.1 (Measurement)**.** A measurement is understood as an operational procedure that yields a numerical representation of a positional attribute.

Throughout this work, numerical representations are expressed in positional decimal notation. Accordingly, a finite numerical representation means a finite positional decimal representation.

Under the assumptions introduced below, we show that this numerical representation is necessarily finite.

This finite representation is not assumed to reflect a discretization of the ontic position space but rather constitutes the output of a numerical encoding map associated with the measurement process.

Accordingly, measurements do not directly yield ontic positions themselves, nor arbitrary elements of $\mathbb{R}$. Instead, they yield finite numerical representations that can subsequently be interpreted within an appropriate mathematical model as representations of ontic positions.

Definition 3.2 (Primary Measurement Result)**.** A primary measurement result is understood as the immediate outcome of a measurement process before any further mathematical operations, model assumptions, or theoretical interpretations are applied.

A primary measurement result is therefore the information that is directly available to an observer after completion of the measurement and can be physically stored, displayed, or communicated.

Position and length measurements belong to the class of primary measurements. For example, when the position of a localized physical entity is reported as a numerical value, this reported value itself constitutes the primary measurement result.

By contrast, quantities such as velocity, acceleration, density, electric power, or other derived observables generally arise only through mathematical processing of one or more primary measurement results and therefore constitute secondary quantities.

Throughout the remainder of this work, the term measurement result always refers to a primary measurement result unless explicitly stated otherwise.

Lemma 3.3. *Every primary measurement result necessarily possesses a finite description length.*

In particular, no primary measurement result can consist of an infinite decimal expansion.

Proof. Let R denote the primary result of an arbitrary measurement process.

By Definition 3.2, R is the information that becomes directly available to an observer immediately after completion of the measurement. Consequently, R must possess a physical representation, for example as the state of a memory device, a detector output, a display, a written record, or a communication signal.

Every completed physical representation necessarily consists of a finite sequence of symbols. Otherwise, the complete measurement result could neither be stored, completely displayed, nor completely transmitted to an observer.

Assume, for contradiction, that the complete decimal expansion of R itself forms part of the primary measurement result and contains infinitely many digits. Then the physical representation of R would necessarily require an infinite number of symbols. Such a representation could never exist as a completed physical record and therefore could never become completely available to an observer.

This contradicts the definition of a primary measurement result.

The conclusion is independent of the technical performance of measuring devices. Even an idealized measuring apparatus with infinite spatial resolution could at most distinguish arbitrarily small positional differences. It would still produce a primary measurement result only after the acquired information has been physically represented. Since every physical representation necessarily contains finitely many symbols, infinite spatial resolution does not imply an infinitely long measurement result.

Hence, no primary measurement result can contain an infinite decimal expansion as part of its physical representation.

Therefore, every primary measurement result necessarily possesses a finite description length. □

3.1 Representation Independence of Measurement Resolution

The preceding lemma does not rely on any technical limitation of present-day measuring instruments.

One might argue that an idealized measuring device possessing infinite spatial resolution could determine positional values with arbitrarily high precision. Even if such an idealization is admitted, the conclusion of Lemma 3.3 remains unchanged.

Infinite spatial resolution merely allows arbitrarily small positional differences to be distinguished. It does not imply that the resulting measurement output itself becomes available as an infinite decimal expansion.

For a measurement result to exist, the acquired information must become physically accessible to an observer. This necessarily requires a finite physical representation. An infinite decimal expansion would require an infinite number of symbols and therefore could neither be completely stored, completely displayed, nor completely communicated.

Consequently, the restriction established in Lemma 3.3 follows from the concept of a completed measurement result itself and not from the finite resolution of measuring devices.

Even an idealized measuring apparatus with infinite spatial resolution can therefore produce only finitely encoded primary measurement results.

3.2 Coordinate Freedom

The numerical value assigned to a measured position depends on the choice of coordinate system used to represent the underlying ontic position.

The choice of coordinate origin and length unit constitutes a purely representational convention and possesses no physical significance. It neither alters the ontic position itself nor affects the outcome of the physical measurement process.

Throughout this work, the coordinate origin and length unit may therefore be chosen freely whenever this simplifies the numerical representation of exact measurement results.

Proposition 3.4 (Coordinate Freedom)**.** *For every fundamentally localizable entity, one can always choose a coordinate origin such that the corresponding exact position measurement possesses a finite numerical representation.*

Proof. Let x denote the ontic position of a fundamentally localizable entity. Since there exists a finitely representable number f, there exists an affine coordinate transformation

$$x' = \kappa x + \lambda, \qquad \kappa \neq 0.$$

Indeed, choosing

$$\lambda = f - \kappa x,$$

yields

$$x' = f.$$

Hence, every fixed ontic position admits an exact finite numerical representation in a suitably chosen coordinate system.

Consequently, the occurrence of an infinite decimal expansion is not an invariant property of the physical position itself but solely of the chosen numerical representation. Therefore, every independently localizable entity admits an exact finite numerical representation as a primary measurement result. □

Example 3.5 (Choice of Length Unit). Consider two idealized measuring devices observing the same localizable entity from a common coordinate origin.

Suppose the first device employs the length unit A, whereas the second device uses the unit B, related by

$$3A = 1B.$$

Assume that the entity is located exactly at the position $1A$. The first measuring device therefore produces the exact primary measurement result $1A$. The second measuring device observes the identical physical position but must represent it numerically as

$$\frac{1}{3}B.$$

Since $0.333\ldots$ has no finite decimal representation, two possibilities remain: Either the measurement process terminates after finitely many digits, yielding only an approximation of the exact position, or the measurement process never produces a completed numerical result. Neither possibility provides an exact primary measurement result in the chosen unit.

This example illustrates that the finiteness of a numerical representation depends on the chosen unit, whereas the underlying ontic position remains unchanged.

Accordingly, throughout the subsequent analysis, exact primary measurement results are represented in units chosen such that their numerical representations are finite.

Approximate measurement results are excluded from the subsequent analysis because the no-go theorem concerns only exact positional assignments.

3.3 Measurement Back-Action

Operationally, a measurement involves a physical interaction through which information about the system is obtained. Throughout this work, we restrict our attention to exact position measurements. For such measurements, the interaction with the measuring apparatus induces a non-vanishing change in the system's position, which we refer to as the measurement back-action.

No assumptions are made regarding the physical origin, magnitude, or dynamical mechanism of the measurement back-action. The only assumptions are that the measurement process possesses a well-defined initial and final ontic position and that the induced positional change satisfies

$$x_{\text{ba}} \neq 0.$$

Let x denote the initial ontic position immediately before the measurement interaction and $\bar{x}$ the final ontic position immediately after completion of the measurement. The measurement back-action is denoted by x_{ba}. It represents the positional change induced by the measurement interaction itself. Consequently, every exact position measurement satisfies the fundamental relation

$$\bar{x} = x + x_{\mathrm{ba}}. \tag{3.1}$$

where

- x denotes the initial ontic position,
- $\bar{x}$ denotes the final ontic position,
- $x_{\mathrm{ba}} \neq 0$ denotes the measurement back-action.

Equation (3.1) constitutes the fundamental measurement equation underlying the remainder of this work.

3.4 Exact Numerical Representation of the Fundamental Measurement Equation

According to Lemma 3.3, every primary measurement result possesses a finite numerical representation.

The initial position x therefore admits a finite numerical representation.

The same conclusion holds for the remaining two quantities appearing in Eq. (3.1).

To see this, consider two exact position measurements performed consecutively on the same localizable entity.

For the first measurement,

$$\bar{x}_1 = x_1 + x_{\mathrm{ba},1}.$$

Immediately afterwards, a second exact position measurement yields

$$\bar{x}_2 = x_2 + x_{\mathrm{ba},2}.$$

Since the second measurement starts precisely where the first measurement ends,

$$x_2 = \bar{x}_1.$$

By Lemma 3.3, the primary measurement result x_2 possesses a finite numerical representation.

Hence the final position $\bar{x}_1$ must also possess a finite numerical representation.

Since this argument is independent of the particular measurement under consideration, every final ontic position appearing in Eq. (3.1) necessarily admits a finite numerical representation.

Because both $\bar{x}$ and x possess finite numerical representations, the measurement back-action must likewise admit a finite numerical representation by Eq. (3.1).

The preceding observations immediately imply the following lemma.

Lemma 3.6. *When expressed in the unit associated with the corresponding primary measurement result, every exact position measurement satisfies a representation of Eq.* (3.1) *in which*

- *the initial ontic position,*
- *the final ontic position,*
- *and the measurement back-action*

all possess finite numerical representations.

Proof. The initial position is finite by Lemma 3.3.

The finiteness of the final position follows from the argument based on two consecutive exact measurements established above.

The measurement back-action is obtained as the difference of two finitely represented quantities according to Eq. (3.1).

Therefore, all three quantities admit finite numerical representations. □

Definition 3.7 (Physical Measurement Equation)**.** An exact position measurement represented in the unit associated with its corresponding primary measurement result is called a *physical measurement equation* if all three quantities appearing in Eq. (3.1),

$$\bar{x},\ x,\ x_{\mathrm{ba}},$$

possess finite numerical representations.

Only under this condition can all numerical values occurring in Eq. (3.1) arise directly as exact primary measurement results.

A mathematically equivalent representation obtained, for example, by a change of length unit may contain one or more quantities with infinite decimal expansions.

Such representations are referred to as *unphysical measurement equations*, since at least one of the quantities $\bar{x}$, x, or x_{ba} is represented by a numerical value that can no longer occur as an exact primary measurement result but arise only through subsequent mathematical transformations.

3.5 Arithmetic Representation of Physical Measurement Equations

From this point onward, we consider only physical measurement equations in the sense of Definition 3.7.

By Lemma 3.6, each quantity appearing in the fundamental measurement equation (3.1) possesses a finite numerical representation. Consequently, every term belongs to the class of finitely representable rational numbers. The remainder of the proof therefore concerns exclusively the arithmetic structure of such representations.

The decimal system is used throughout this work solely as an illustrative example. All subsequent arguments remain valid in any positional numeral system after replacing the corresponding prime factors of the base.

In the decimal system, every finitely representable number possesses a unique reduced fractional representation whose denominator contains no prime factors other than 2 and 5.

Accordingly, the three quantities appearing in Eq. (3.1) necessarily admit representations of the form

$$\bar{x} = \frac{a}{2^i 5^j}, \tag{3.2}$$
$$x = \frac{c}{2^k 5^l}, \tag{3.3}$$
$$x_{\mathrm{ba}} = \frac{e}{2^m 5^n}, \tag{3.4}$$

where

$$a, c, e \in \mathbb{Z} \setminus \{0\},$$

$$i, j, k, l, m, n \in \mathbb{Z},$$

and

$$\gcd(a, 2^i 5^j) = \gcd(c, 2^k 5^l) = \gcd(e, 2^m 5^n) = 1.$$

Negative exponents are permitted and represent factors of 2 or 5 in the numerical value that are not contained in the numerators a, c, and e. Thus, the above expressions describe all finitely representable rational numbers.

Substituting Eqs. (3.2)–(3.4) into the fundamental measurement equation, Eq. (3.1), yields

$$\frac{a}{2^i 5^j} = \frac{c}{2^k 5^l} + \frac{e}{2^m 5^n}. \tag{3.5}$$

Equation (3.5) constitutes the canonical arithmetic representation of every exact physical position measurement.

From this point onward, the proof no longer depends on the physical interpretation of the three quantities but only on the arithmetic properties of Eq. (3.5).

All subsequent arguments used in the proof of the no-go theorem are therefore purely number-theoretic.

3.6 Choice of Coordinate Origin

The numerical values appearing in Eq. (3.5) depend on the arbitrary choice of coordinate origin.

Since the coordinate origin possesses no physical significance, it may be chosen freely to eliminate unnecessary arithmetic degeneracies.

Throughout the remainder of this work, the coordinate origin is therefore selected according to the following conditions.

Condition 3.8. The coordinate origin is chosen such that

$$x \neq 0, \qquad \bar{x} \neq 0.$$

This excludes trivial two-term representations of Eq. (3.5).

Since the position of the coordinate origin is entirely conventional, this restriction has no physical consequences.

Condition 3.9. The coordinate origin is furthermore chosen such that

$$|a| \neq |c|, \qquad |a| \neq |e|, \qquad |c| \neq |e|.$$

If one of these equalities occurs, the coordinate origin can always be translated by a suitably chosen finite displacement. Such a translation changes only the numerical representation of the measurement while leaving the underlying physical measurement unchanged. Consequently, these equalities may always be excluded without loss of generality.

Note that a coordinate displacement adds the same numerical value to both $\bar{x}$ and x. The measurement back-action x_{ba}, being the fixed positional difference between $\bar{x}$ and x, therefore remains unchanged, since $x_{\mathrm{ba}} = \bar{x} - x$.

Lemma 3.10. *Equation* (3.5) *may always be assumed to be in maximally reduced form, i.e.,*

$$\gcd(a, c) = 1, \qquad \gcd(a, e) = 1, \qquad \gcd(c, e) = 1.$$

Proof. If all three numerators share a common factor distinct from the prime factors 2 and 5, this factor may be cancelled simultaneously from all three fractions without affecting Eq. (3.5).

Suppose, instead, that only two of the three numerators in Eq. (3.5) possess a common factor

$$y \neq 2^r 5^s.$$

Consequently, the third numerator is coprime to y.

Cancelling this factor from only two fractions necessarily introduces the factor y into the denominator of the remaining fraction. Since y contains a prime factor different from 2 or 5, the remaining fraction would no longer possess a finite decimal representation. This contradicts Lemma 3.6.

Hence Eq. (3.5) may always be assumed to satisfy

$$\gcd(a, c) = \gcd(a, e) = \gcd(c, e) = 1.$$

□

Equation (3.5), together with Lemma 3.10 and Conditions 3.8–3.9, completely specifies the arithmetic structure of every exact physical position measurement considered in this work. All subsequent results follow exclusively from elementary properties of this arithmetic structure.

3.7 Representation Equivalence under Unit Transformations

The arithmetic structure derived above is independent of the particular unit of length chosen for its numerical representation.

Mathematically, a unit conversion corresponds to a linear scaling

$$x_{\mathrm{conv}} = b\, x_{\mathrm{orig}},$$

where

$$b$$

denotes the conversion factor.

A purely mathematical unit conversion imposes no restriction on the resulting numerical values.

Physical measurements, however, require additional consistency conditions.

Whether a converted representation remains physically admissible depends on the numerical representation of the converted quantities rather than on the scaling operation itself.

Definition 3.11 (Original and Converted Measurement Equations)**.** A physical measurement equation obtained directly from an exact measurement is called the *original measurement equation*.

Since every primary measurement result possesses a finite numerical representation, every original measurement equation is necessarily physical.

A converted measurement equation is obtained from the original measurement equation by a unit transformation.

The converted equation is called

- *physical*, if all three quantities

$$\bar{x},\ x,\ x_{\mathrm{ba}}$$

  remain finitely representable,

- *unphysical*, if at least one of the three quantities no longer possesses a finite numerical representation and therefore cannot occur as an exact primary measurement result.

Lemma 3.12. *Every finitely representable numerical value can occur individually as the numerical representation of the final position $\bar{x}$, the initial position x, and the measurement back-action x_{ba} in an exact physical measurement.*

Consequently, the admissible numerical representations of the final position, the initial position, and the measurement back-action are all given by the common set

$$D = \left\{ \frac{p}{2^g 5^h} \;\middle|\; p \in \mathbb{Z} \setminus \{0\},\ g, h \in \mathbb{Z} \right\},$$

that is,

$$\bar{x}, x, x_{\mathrm{ba}} \in D,$$

and

$$D_{\bar{x}} = D_x = D_{x_{\mathrm{ba}}} = D.$$

This statement holds equally for original and physical converted measurement equations. Therefore,

$$D_{\mathrm{orig}} = D_{\mathrm{conv}} = D.$$

Proof. Consider an original physical measurement equation in the sense of Definition 3.11, obtained with a measuring device whose native unit is A.

Now suppose this equation is converted into another unit B while remaining physical.

Can the converted equation itself be reproduced experimentally?

The answer is affirmative.

One simply performs the measurement with a measuring device whose native unit is B.

By the preceding measurement framework and the idealized localization limit established in the previous section, all restrictions originating from finite detector resolution and uncertainty-related limitations are removed. Consequently, finite representability itself is the only remaining structural restriction on admissible numerical values. The attainable numerical representations are therefore determined solely by the chosen physical configuration of the measurement process, whose initial conditions, boundary conditions, detector realization, and interaction mechanism may be chosen freely

Since the measurement back-action is determined by the complete experimental realization, the numerical representations of all three quantities may therefore be chosen independently by an appropriate choice of the measurement configuration.

Every finitely representable numerical value can therefore occur independently as the numerical representation of each of the three quantities.

It follows immediately that all three quantities possess the same admissible numerical set,

$$D_{\bar{x}} = D_x = D_{x_{\text{ba}}} = D.$$

Moreover, every physical converted measurement equation can itself be reproduced as an original physical measurement equation by choosing a measuring device whose native unit coincides with the converted unit.

Conversely, every original physical measurement equation may serve as the starting point of a physical unit conversion without restrictions on the set of admissible numerical values.

Therefore, every admissible numerical representation occurring in an original physical measurement equation also occurs in a physical converted measurement equation, and vice versa.

Consequently, both equations possess identical admissible numerical sets,

$$D_{\text{orig}} = D_{\text{conv}} = D.$$

Hence, the admissible numerical set is determined by the measurement framework itself rather than by any particular measurement realization, measuring device, or native length unit. Consequently, every physical unit conversion preserves this admissible numerical set, and any physical converted measurement equation may therefore be used interchangeably with the corresponding original measurement equation with respect to admissible numerical representations.

□

The proof of Lemma 3.12 establishes the equality

$$D_{\text{orig}} = D_{\text{conv}} = D.$$

The following two sections are not part of the proof itself. Instead, they clarify the physical meaning and scope of this result.

The operational interpretation explains why the distinction between original and physically converted measurement equations has no intrinsic physical significance, while the discussion of universality shows why the proven equality is independent of the particular realization of the measurement process.

Together, these discussions show that the equality established in Lemma 3.12 is not merely a property of the specific construction used in the proof but a structural property of the measurement framework itself.

3.8 Operational Interpretation of Lemma 3.12

The previous result also admits a direct operational interpretation: Every measuring apparatus is calibrated with respect to an internal reference length scale. This reference scale may itself originate from a conversion of an earlier physical length standard during the manufacturing process.

Consequently, what is regarded operationally as an original measurement equation may already correspond to a physically converted measurement equation relative to an earlier reference length standard.

For example, a measuring device calibrated in length unit A may have been manufactured using a reference scale that itself originated from a conversion of another length unit A'. In this case, measurements performed in A are already physically converted with respect to A', although they operationally appear as original measurements.

Therefore, no physical criterion exists that uniquely determines which reference length standard should be regarded as the original one.

Thus, the choice of the reference standard is operational rather than an intrinsic property of the physical system.

Consequently, the admissible numerical set associated with a physical measurement (original or converted) cannot depend on the particular reference length standard that is chosen.

3.9 Universality of Lemma 3.12

The equality

$$D_{\mathrm{orig}} = D_{\mathrm{conv}} = D$$

is universal within the measurement framework established above.

Assume, for contradiction, that the admissible numerical set depended on the particular realization of the measurement process, for example on the detector realization, interaction mechanism, boundary conditions, or on the original measurement equation from which a physical conversion is performed.

Then two admissible realizations of the same physical measurement could possess different admissible numerical sets.

This immediately contradicts the equality established in Lemma 3.12.

According to Lemma 3.12, every physical converted measurement equation can itself be realized as an original physical measurement equation.

Consequently, the admissible numerical set must remain unchanged under every physical unit conversion.

Otherwise, a physical converted measurement equation obtained from one realization could no longer be reproduced by another admissible realization.

An even stronger contradiction follows from the previous section.

There we established that the distinction between an original and a physically converted measurement equation is purely operational: it depends only on the choice of reference length standard, not on any intrinsic physical property of the measurement.

Therefore, changing the reference standard must never alter any physical property of the measurement.

If different realizations possessed different admissible numerical sets, then the admissible numerical representations would depend on how the reference length standard had been established.

In other words, the history of the measuring device or the particular realization of the measurement process would determine which numerical representations are physically admissible.

This would make the choice of the reference length standard physically distinguishable.

However, the previous section showed that no reference length standard possesses a privileged physical status.

The admissible numerical set therefore cannot depend on the particular realization of the measurement process.

Therefore,

$$D_{\mathrm{orig}} = D_{\mathrm{conv}} = D$$

is necessarily universal.

3.10 Concluding Remark on the Representation Framework

The purpose of the preceding construction was not to derive a particular physical model of measurements but to identify the arithmetic structure shared by all exact physical position measurements satisfying the axioms introduced in Section 2.

The measurement framework establishes that every exact physical position measurement admits a canonical arithmetic representation of the form given by Eq. (3.5). This representation is independent of the particular measuring device, experimental realization, physical interaction mechanism, or dynamical theory.

Furthermore, the construction neither requires nor implies any discreteness of the underlying ontic position space. The restriction to finite numerical representations concerns exclusively the numerical encoding of exact measurement results.

It is important to emphasize that Lemma 3.6 pertains solely to the numerical representation of measured positions. It neither imposes any discreteness of space itself nor assumes the existence of localized physical objects beyond the axioms introduced previously.

Consequently, the contradiction derived in the following sections ultimately originates from the arithmetic structure of Eq. (3.5) together with the elementary properties of finite numerical representations. No additional physical assumptions enter the argument.

4 Main Result

According to Lemma 3.12, every finitely representable numerical value belonging to D may occur independently as the numerical representation of the final ontic position $\bar{x}$, the initial ontic position x, and the measurement back-action x_{ba} in an exact physical measurement equation. This statement holds equally for original and physical converted measurement equations. Consequently, every finitely representable decimal must be capable of appearing in each of the three quantities individually.

Let

$$\frac{q}{2^o 5^p}$$

be an arbitrary finite decimal satisfying

$$\gcd(q, 2^o 5^p) = 1,$$

where

$$q \in \mathbb{Z} \setminus \{0\}, \qquad o, p \in \mathbb{Z}.$$

Since every converted measurement equation originates from an original physical measurement equation, we use the arithmetic measurement Eq. (3.5) as the original equation and investigate whether the arbitrary finite decimal

$$\frac{q}{2^o 5^p}$$

can become the numerical representation of the final ontic position, the initial ontic position, and the measurement back-action in three different physical converted measurement equations.

Throughout the following derivation, it must be kept in mind that the integers a, c, e, i, j, k, l, m, and n appearing in the arithmetic representation of Eq. (3.5) are measurement-specific quantities. Different measurements, different detector resolutions, modified interaction mechanisms or different boundary conditions generally lead to different values of these parameters. The subsequent argument must therefore hold for any choice of the original measurement equation; in particular, it must be independent of the specific numerical values of a, c, e, i, j, k, l, m, n.

We first require

$$\frac{q}{2^o 5^p}$$

to become the numerical representation of the final ontic position.

Dividing Eq. (3.5) by

$$\bar{x} = \frac{a}{2^i 5^j}$$

and multiplying the resulting equation by

$$\frac{q}{2^o 5^p}$$

corresponds to a unit conversion with conversion factor

$$\frac{q}{2^o 5^p} \cdot \frac{2^i 5^j}{a}.$$

The quantities in the converted measurement equation are indexed by c_1.

This yields

$$\underbrace{\frac{q}{2^o 5^p}}_{\bar{x}_{(c_1)}} = \underbrace{\frac{qc}{a\,2^{\,k-i+o}5^{\,l-j+p}}}_{x_{(c_1)}} + \underbrace{\frac{qe}{a\,2^{\,m-i+o}5^{\,n-j+p}}}_{x_{\mathrm{ba}(c_1)}}\,. \tag{4.1}$$

By construction, the final ontic position already possesses the desired finite representation. For Eq. (4.1) to remain a physical converted measurement equation, both remaining quantities must likewise possess finite numerical representations.

Lemma 3.10 established that

$$\gcd(a, c) = 1$$

and

$$\gcd(a, e) = 1.$$

Hence, the only possible obstruction to finite representability of the two remaining quantities is the factor a occurring in the denominator. Therefore Eq. (4.1) remains physical if and only if

$$\frac{q}{a} \in \mathbb{Z} \setminus \{0\}. \tag{4.2}$$

Equation (4.2) therefore provides the necessary and sufficient arithmetic condition under which the arbitrary finite decimal

$$\frac{q}{2^o 5^p}$$

can occur as the numerical representation of a final ontic position in a physical converted measurement equation.

An important observation already emerges at this stage. Unlike a purely mathematical scaling, a physical unit conversion is not automatically admissible. It is subject to an arithmetic consistency condition. Whenever condition (4.2) is violated, the converted equation still exists as a mathematical identity but ceases to represent a physical measurement equation because one or more quantities no longer possess finite numerical representations.

Condition (4.2) already reveals the first manifestation of the contradiction. In order for the converted measurement equation to remain physical, the admissible numerical representations are no longer arbitrary but become restricted by the measurement-specific parameter a. Consequently, the admissible numerical set associated with this converted equation is no longer guaranteed to coincide with the unrestricted set D established in Lemma 3.12.

At this stage, however, the restriction still depends on the particular measurement through the parameter a. It therefore cannot yet establish a universal contradiction. To determine whether this restriction reflects merely a measurement-specific feature or a structural property of the measurement framework itself, the same analysis must be repeated for the remaining two quantities x and x_{ba}.

We now require the same arbitrary finite decimal

$$\frac{q}{2^o 5^p}$$

to become the numerical representation of the initial ontic position. The procedure is completely analogous. We divide Eq. (3.5) by

$$x = \frac{c}{2^k 5^l}$$

and multiply by

$$\frac{q}{2^o 5^p}.$$

The converted quantities are indexed by c_2, yielding

$$\underbrace{\frac{qa}{c\,2^{\,i-k+o}5^{\,j-l+p}}}_{\bar{x}_{(c_2)}} = \underbrace{\frac{q}{2^o 5^p}}_{x_{(c_2)}} + \underbrace{\frac{qe}{c\,2^{\,m-k+o}5^{\,n-l+p}}}_{x_{\mathrm{ba}(c_2)}}. \tag{4.3}$$

Again, the initial ontic position already possesses the desired finite numerical representation by construction. The converted equation remains physical only if both remaining quantities continue to admit finite decimal representations.

From Lemma 3.10 we have

$$\gcd(a, c) = 1$$

and

$$\gcd(c, e) = 1.$$

Therefore the only possible obstruction is the denominator c. Consequently, Eq. (4.3) remains physical if and only if

$$\frac{q}{c} \in \mathbb{Z} \setminus \{0\}. \tag{4.4}$$

Equation (4.4) is therefore the necessary and sufficient arithmetic condition under which the arbitrary finite decimal

$$\frac{q}{2^o 5^p}$$

can occur as the numerical representation of an initial ontic position in a physical converted measurement equation.

Lemma 3.10 guarantees that a and c are coprime,

$$\gcd(a, c) = 1,$$

while Condition 3.9 requires their absolute values to be distinct,

$$|a| \neq |c|.$$

Thus, combining Eqs. (4.2) and (4.4) gives

$$\frac{q}{ac} \in \mathbb{Z} \setminus \{0\}.$$

Since a and c are coprime and $|a| \neq |c|$, they cannot both have absolute value one. Hence $|ac| > 1$. It follows that ac cannot divide $q = \pm 1$, and therefore

$$q = \pm 1,$$

which is the measurement-independent restriction that we were looking for. However, this result is not yet fully symmetric with respect to the three quantities $\bar{x}$, x, and x_{ba}. The remaining conversion is therefore required to establish the complete structural restriction without privileging any particular term of the measurement equation.

Hence, we finally require the same arbitrary finite decimal

$$\frac{q}{2^o 5^p}$$

to become the numerical representation of the measurement back-action. Dividing Eq. (3.5) by

$$x_{\mathrm{ba}} = \frac{e}{2^m 5^n}$$

and multiplying by

$$\frac{q}{2^o 5^p}$$

produces a third converted measurement equation, whose quantities are indexed by c_3:

$$\underbrace{\frac{aq}{e\,2^{\,i-m+o}5^{\,j-n+p}}}_{\bar{x}_{(c_3)}} = \underbrace{\frac{cq}{e\,2^{\,k-m+o}5^{\,l-n+p}}}_{x_{(c_3)}} + \underbrace{\frac{q}{2^o 5^p}}_{x_{\mathrm{ba}(c_3)}}. \tag{4.5}$$

Since the measurement back-action already possesses the desired finite representation, physical admissibility requires only that the remaining two quantities remain finitely representable.

Applying Lemma 3.10 once more,

$$\gcd(a, e) = 1$$

and

$$\gcd(c, e) = 1,$$

shows that this is equivalent to the condition

$$\frac{q}{e} \in \mathbb{Z} \setminus \{0\}. \tag{4.6}$$

Equation (4.6) is therefore the necessary and sufficient arithmetic condition under which the arbitrary finite decimal

$$\frac{q}{2^o 5^p}$$

can occur as the numerical representation of the measurement back-action in a physical converted measurement equation.

Including the third conversion completes the symmetry of the argument. At this point the three conversion procedures have established the complete set of arithmetic conditions that every arbitrary finite decimal must satisfy in order to remain physically admissible. Since, according to Lemma 3.12, every finitely representable numerical value must be capable of occurring independently as a final ontic position, an initial ontic position, and a measurement back-action, conditions (4.2), (4.4), and (4.6) must therefore hold simultaneously for every admissible finite decimal.

Since the integers a, c, and e are pairwise coprime by Lemma 3.10, conditions (4.2), (4.4), and (4.6) can be combined into a single arithmetic condition,

$$\frac{q}{ace} \in \mathbb{Z} \setminus \{0\}. \tag{4.7}$$

Condition (4.7) appears to depend on the particular original measurement through the measurement-specific integers a, c, and e, but this dependence disappears once again by taking into account Lemma 3.10, which guarantees that a, c, and e are pairwise coprime,

$$\gcd(a, c) = \gcd(a, e) = \gcd(c, e) = 1,$$

and Condition 3.9, which requires that their absolute values are pairwise distinct,

$$|a| \neq |c|, \qquad |a| \neq |e|, \qquad |c| \neq |e|.$$

Consequently, condition (4.7) requires the numerator q to possess three pairwise coprime divisors whose absolute values are mutually distinct. Any integer that does not admit such a divisor structure is excluded. Positive and negative prime powers, as well as ± 1, are immediate examples that will be analyzed below.

We first examine the possibility $q = \pm 1$. Since every divisor of ± 1 has absolute value one, condition (4.7) would imply

$$|a| = |c| = |e| = 1,$$

which immediately violates Condition 3.9. Therefore,

$$q \neq \pm 1. \tag{4.8}$$

Second, suppose q is a positive or negative prime power,

$$q = \pm d^f,$$

where d is a prime number ($d \in \mathbb{P}$) and f is a positive integer.

Every non-trivial divisor of a prime power is itself a power of the same prime d. Consequently, any two non-trivial divisors necessarily possess the common prime factor d and therefore cannot be coprime. Condition (4.7), however, requires the existence of three pairwise coprime divisors a, c, and e. Hence, at most one of these divisors can be non-trivial. The remaining divisors must therefore have absolute value one. This is impossible because Condition 3.9 simultaneously requires the absolute values of a, c, and e to be pairwise distinct. Accordingly, q cannot be a positive or negative prime power.

We therefore conclude that

$$q \neq \pm d^f \tag{4.9}$$

for every prime number d and every positive integer f.

The restrictions Eqs. (4.8) and (4.9) already arise in the idealized localization limit. Their origin cannot be attributed to finite detector resolution, uncertainty relations, imperfect measuring devices, or any other physical approximation. They follow exclusively from the assumption that ontic positions admit a consistent numerical representation together with the arithmetic structure established by Eq. (3.5), Lemma 3.10, Lemma 3.12, and Condition 3.9. More importantly, restrictions (4.8) and (4.9) are completely independent of any measurement-specific quantities a, c, e, i, j, k, l, m, and n; they hold for every conceivable measurement configuration.

An important consequence follows immediately. Every finite decimal whose numerator satisfies

$$q = \pm 1$$

or

$$q = \pm d^f$$

is excluded from the set of admissible numerical representations of physical converted measurement equations. Consequently, the admissible set of numerical values for physical converted measurement equations is strictly smaller than the admissible set for original physical measurement equations. However, Lemma 3.12 established that original and physical converted measurement equations possess exactly the same set of admissible numerical representations. In particular,

$$D_{\text{orig}} = D_{\text{conv}} = D. \tag{4.10}$$

Equation (4.10) expresses the invariance of admissible numerical representations under all physical unit conversions. Every finite decimal that can occur in an original physical measurement equation must also occur in a physical converted measurement equation, and conversely.

The arithmetic restrictions derived above contradict this result directly. Eqs. (4.8)–(4.9) exclude infinitely many finite decimals from D_{conv}, namely all finite decimals whose numerators are equal to ± 1 or to a positive or negative prime power. Consequently,

$$D_{\text{orig}} \supsetneq D_{\text{conv}}. \tag{4.11}$$

Eqs. (4.10) and (4.11) are mutually incompatible.

The contradiction cannot be attributed to finite measurement precision, detector resolution, uncertainty-related limitations, or to the particular realization of the measurement process.

Every physical assumption enters the derivation exclusively through the four axioms of this paper. The subsequent contradiction does not rely on any additional physical assumptions. Instead, it follows solely from the arithmetic structure of the measurement equation together with Lemma 3.10, Lemma 3.12, Condition 3.8, Condition 3.9, and elementary divisibility properties. The contradiction therefore originates solely from the assumption that ontic positions admit a consistent injective numerical representation. Thus, the assumption must be false.

Hence, the Ontic Position Hypothesis cannot be consistently maintained under the stated axioms.

The argument is independent of the numerical knowledge of the measurement-specific quantities a, c and e. These quantities enter only as arbitrary numerical representations of the respective physical quantities, and their particular values are not required at any stage of the derivation. The resulting contradiction is therefore independent of the particular measurement values and, in this sense, measurement independent.

The robustness of the result against measurement uncertainties, perturbations of the coordinate description, and alternative descriptions in terms of spatially extended entities or field excitations is discussed in Appendix B.

One may object that the above derivation starts from a single original measurement equation and generates three physical converted measurement equations from it. This does not restrict the generality of the proof. By Lemma 3.12, the admissible numerical set is identical for every original and every physical converted measurement equation, independently of the original measurement from which the conversion is performed. Consequently, replacing the single original measurement equation by three independent original measurement equations leaves the admissible numerical set unchanged and cannot remove the contradiction. The corresponding formal derivation is given in Appendix A. The result remains unchanged.

A further class of objections concerns the distinction between an ontically defined position and its experimental determination. One may object that the position entering the measurement equation cannot be determined exactly because of measurement uncertainty, including the Heisenberg uncertainty relation. Related objections concern perturbations of the coordinate system or replacing a pointlike entity by a spatially extended structure. These possibilities do not alter the logical structure of the result. As shown in Appendix B, measurement uncertainty can be represented by an additional unknown positional contribution, while coordinate perturbations and spatial extension do not remove the contradiction as long as the fundamental entity retains an ontically defined spatial property admitting a numerical representation.

5 Conclusion

5.1 Physical Implications and Scope of the No-Go Theorem

A natural objection is that the present theorem applies only to ontic positions admitting an injective numerical representation and therefore may leave open the possibility of fundamentally non-representable ontic positions.

Formally, this statement is correct. The theorem excludes only ontic positions that can be embedded into a consistent numerical representation structure.

However, the physical significance of this restriction is considerably stronger than it may initially appear.

Within the conventional formulation of most physical theories, assigning a position to a physical entity is naturally accompanied by a consistent coordinate representation. Once a coordinate assignment exists, an injective numerical representation follows naturally. The assumptions required by the theorem therefore do not arise as artificial mathematical constraints but rather as the standard operational meaning of what physicists ordinarily understand by a position. Consequently, avoiding the present theorem requires more than abandoning a particular numerical representation. It requires abandoning the conventional notion of position addressed in this work. Any ontology that retains position as a fundamental attribute while allowing consistent coordinate assignment falls within the scope of the present theorem. Escaping the theorem therefore requires adopting a fundamentally different concept of position rather than merely a different mathematical representation.

Consequently, any theory escaping the present no-go theorem must abandon not merely a particular numerical representation but the possibility of assigning fundamental positions through any consistent coordinate-based description.

The remaining class of admissible ontologies therefore appears highly restricted. Such theories would have to abandon fundamental positions as primitively coordinatizable entities and instead formulate their ontology in terms of more abstract structures, for example relational, algebraic, information-theoretic, or emergent frameworks, including algebraic quantum field theory [2–8], in which position may no longer be regarded as a primitive ontological property.

5.2 Interpretation of the Result

From this perspective, the theorem does not merely constrain a particular class of hidden-variable theories or ontological particle models [9].

Instead, it suggests a more general conclusion: if the assumptions developed in this work are accepted, fundamental physical entities cannot possess ontic positions in the conventional sense of uniquely assignable coordinate locations.

Any viable ontology must therefore either abandon ontic position as a fundamental property or explain why its notion of position does not satisfy the assumptions of the present theorem.

The same observation extends to physical quantities defined through exact spatial localization, such as geometrical lengths or characteristic spatial scales. These quantities likewise inherit the arithmetic obstruction whenever they rely on fundamentally well-defined positional assignments.

Importantly, the theorem does not exclude emergent or effective notions of position. It constrains only the possibility that position constitutes a fundamental ontological property admitting a consistent numerical representation.

The result is therefore compatible with approaches in which position arises only at an effective or emergent level, while the underlying ontology is formulated in terms of more abstract structures.

The contradiction is likewise unaffected by stochastic perturbations, modified coordinate systems, or alternative deterministic realizations. Such modifications may change the particular numerical representation associated with a physical measurement but do not alter the underlying assumptions of the theorem. As long as position remains a fundamental ontological property admitting a consistent numerical representation, the arithmetic argument developed in this work applies unchanged.

5.3 Broader Consequences

The significance of the theorem therefore extends beyond the specific assumptions used in its derivation.

The contradiction obtained in this work originates from the combination of Axioms 2.3–2.6 introduced earlier with the arithmetic structure of exact positional assignments. Once a fundamental entity is assumed to possess a uniquely identifiable position that admits a consistent numerical representation, the resulting structure leads to the inconsistency demonstrated by the no-go theorem.

Consequently, the set of ontologies capable of avoiding the theorem appears substantially smaller than the formulation “injectively representable ontic positions” may initially suggest. While logically conceivable alternatives remain possible, they must dispense with the assumption of a fundamentally defined and numerically representable position, or employ a different ontological structure to which the assumptions of the theorem do not apply.

The theorem therefore constrains the possible ontology of a fundamental particle. A viable fundamental description must either abandon an ontically defined numerical position or employ a different ontological structure to which the assumptions of the theorem do not apply. The theorem itself does not determine which alternative description, if any, provides the correct fundamental account.

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A Independence of the Original Measurement Equation

The main proof derives the contradiction by starting from a single original physical measurement equation and generating three physical converted measurement equations from it. By Lemma 3.12, this choice does not restrict the generality of the argument, since every original and every physical converted measurement equation possesses the same universal admissible numerical set D, independently of the particular original measurement equation. The present appendix nevertheless considers the more general case of three independent original measurement equations and demonstrates explicitly that the contradiction remains unchanged.

Suppose that three independent original physical measurement equations are given by

$$\frac{a_1}{2^{i_1}5^{j_1}} = \frac{c_1}{2^{k_1}5^{l_1}} + \frac{e_1}{2^{m_1}5^{n_1}}. \tag{A.1}$$

$$\frac{a_2}{2^{i_2}5^{j_2}} = \frac{c_2}{2^{k_2}5^{l_2}} + \frac{e_2}{2^{m_2}5^{n_2}}. \tag{A.2}$$

$$\frac{a_3}{2^{i_3}5^{j_3}} = \frac{c_3}{2^{k_3}5^{l_3}} + \frac{e_3}{2^{m_3}5^{n_3}}. \tag{A.3}$$

where each equation possesses its own measurement-specific integers indicated by the subscripts 1, 2, and 3, respectively.

Each original measurement equation satisfies the assumptions established in the previous sections. Consequently, Lemma 3.10 applies independently to every measurement equation,

$$\gcd(a_i, c_i) = \gcd(a_i, e_i) = \gcd(c_i, e_i) = 1, \qquad i = 1, 2, 3. \tag{A.4}$$

Furthermore, Condition 3.9 requires

$$|a_i| \neq |c_i|, \qquad |a_i| \neq |e_i|, \qquad |c_i| \neq |e_i|. \tag{A.5}$$

Suppose now that the same arbitrary finite decimal

$$\frac{q}{2^o 5^p}$$

is required to occur as a final ontic position in the first converted equation, an initial ontic position in the second converted equation, and a measurement back-action in the third converted equation.

Repeating the derivation of the main proof for the three independent original equations yields

$$\frac{q}{a_1} \in \mathbb{Z} \setminus \{0\}, \tag{A.6}$$
$$\frac{q}{c_2} \in \mathbb{Z} \setminus \{0\}, \tag{A.7}$$
$$\frac{q}{e_3} \in \mathbb{Z} \setminus \{0\}. \tag{A.8}$$

The three integers a_1, c_2, and e_3 need not initially be pairwise coprime since they originate from three independent measurements. If this is the case, the coordinate origins can be translated by a suitably chosen finite displacement to change the numerical values of a_1 and c_2. This is the same logic that underlies Condition 3.9. Under a translation of the coordinate origin, the measurement back-action x_{ba}, and hence e_3, remains unchanged, since it is simply the fixed positional difference between the final and initial positions, $x_{\text{ba}} = \bar{x} - x$. Such a finite displacement preserves the physicality of the measurement since all three quantities $\bar{x}$, x, and x_{ba} remain finite decimals in their respective equations.

Example (Coordinate Translation). Consider a physical measurement equation of the form

$$\bar{x} = x + x_{\text{ba}}.$$

Displacing the coordinate origin by the finite decimal x_{disp} leaves the measurement back-action x_{ba} unchanged, since $x_{\text{ba}} = \bar{x} - x$, whereas the same coordinate displacement is added to both the initial and final positions:

$$\underbrace{(\bar{x} + x_{\mathrm{disp}})}_{\bar{x}_{\mathrm{new}}} = \underbrace{(x + x_{\mathrm{disp}})}_{x_{\mathrm{new}}} + x_{\mathrm{ba}}.$$

This equation remains physical. Only the numerical values assigned to the initial and final positions have changed. Hence, a finite displacement of a coordinate origin can modify the relevant numerical values a_1 and c_2 accordingly. Therefore, without loss of generality, the integers a_1, c_2, and e_3 can always be chosen such that

$$\gcd(a_1, c_2) = \gcd(a_1, e_3) = \gcd(c_2, e_3) = 1.$$

Now the three conditions can be combined into

$$\frac{q}{a_1 c_2 e_3} \in \mathbb{Z} \setminus \{0\}. \tag{A.9}$$

From here, the argument of the main theorem applies without modification. The arithmetic restrictions obtained are therefore identical to those derived in the main proof. The contradiction with Lemma 3.12 follows immediately.

The admissible numerical set of physical converted measurement equations cannot depend on the particular original measurement equation from which the conversion is generated. Such a dependence would be incompatible with Lemma 3.12, which establishes the universal equality

$$D_{\mathrm{orig}} = D_{\mathrm{conv}} = D.$$

Accordingly, replacing one original measurement equation by several independent original measurement equations does not weaken or remove the contradiction.

The no-go theorem is therefore independent of the choice of original measurement equation.

B Independence from Measurement and Ontological Modifications

The main result does not depend on the numerical knowledge of the measurement-specific quantities appearing in the construction. Nevertheless, one may object that the positional quantities entering the measurement equation are not exactly accessible because of measurement uncertainties, including uncertainties associated with the Heisenberg uncertainty relation. This appendix shows that introducing such uncertainties does not alter the arithmetic structure responsible for the contradiction.

B.1 Positional Uncertainty and the Heisenberg Relation

Consider an unknown positional contribution Δx between the underlying ontic position and the numerical quantities entering the measurement equation. The physical measurement equation can then be written as

$$\bar{x} = x + x_{\mathrm{ba}} + \Delta x. \tag{B.1}$$

Here, Δx denotes a general unknown positional contribution. It may, for example, represent an uncertainty associated with the measurement process, including an uncertainty motivated by the

Heisenberg uncertainty relation, or any other unknown positional fluctuation or perturbation. The argument below does not depend on the physical origin or numerical knowledge of Δx.

Since $\bar{x}$, x, and x_{ba} belong to the admissible numerical set D, Eq. (B.1) implies that Δx must likewise belong to D. It can therefore be written in the form

$$\Delta x = \frac{g}{2^r 5^s}, \qquad \gcd(g, 2^r 5^s) = 1, \qquad g \in \mathbb{Z} \setminus \{0\}, \qquad r, s \in \mathbb{Z}. \tag{B.2}$$

This requirement does not imply that the value of Δx is known. It only follows from the requirement that the physical measurement equation remains within the admissible numerical set D.

The introduction of Δx does not modify the conversion argument of the main theorem. The same arbitrary finite numerical value

$$\frac{q}{2^o 5^p}, \tag{B.3}$$

may be considered as the target numerical value in each of the three coordinate-conversion procedures used in the main theorem.

In the first conversion, the numerical values of $\bar{x}$, x, and x_{ba} are transformed by the same coordinate-conversion factor as in the original argument. The additional contribution Δx is transformed by the same factor. Its numerator therefore acquires the additional integer factor g, but no new arithmetic condition involving g is required. The condition obtained from the first conversion remains

$$\frac{q}{a} \in \mathbb{Z} \setminus \{0\}. \tag{B.4}$$

Indeed, the corresponding transformed contribution from Δx contains the factor

$$\frac{qg}{a} = \left(\frac{q}{a}\right) g, \tag{B.5}$$

which is an integer whenever Eq. (B.4) holds. No coprimality condition between g and a is required.

The same reasoning applies to the second and third conversions. The corresponding conditions remain

$$\frac{q}{c} \in \mathbb{Z} \setminus \{0\}, \tag{B.6}$$

and

$$\frac{q}{e} \in \mathbb{Z} \setminus \{0\}. \tag{B.7}$$

The additional factor g does not modify either condition, since

$$\frac{qg}{c} = \left(\frac{q}{c}\right) g$$

and

$$\frac{qg}{e} = \left(\frac{q}{e}\right) g$$

are integers whenever Eqs. (B.6) and (B.7) hold, respectively.

Consequently, the three arithmetic conditions obtained in the main theorem remain unchanged. Since a, c, and e are pairwise coprime by Lemma 3.10, they combine to give

$$\frac{q}{ace} \in \mathbb{Z} \setminus \{0\}. \tag{B.8}$$

Thus, the additional positional contribution Δx does not alter the arithmetic condition responsible for the contradiction.

Moreover, no additional condition on the numerical set to which the transformed contribution Δx belongs is required. The derivation of the contradiction depends only on the requirement that the three quantities $\bar{x}$, x, and x_{ba} remain in the admissible numerical set D, as guaranteed by Lemma 3.12. The transformed contribution Δx is therefore irrelevant to the arithmetic conditions generating the contradiction.

Importantly, this conclusion does not depend on numerical knowledge of a, c, e, or g. The argument requires only that the corresponding physical quantities belong to the admissible numerical set D. Consequently, introducing an unknown positional contribution cannot restore consistency. It imposes an additional requirement—namely, that Δx itself belong to D in the original physical measurement equation—but does not remove any of the requirements already responsible for the contradiction.

The Heisenberg uncertainty relation therefore does not provide an independent escape from the theorem. The uncertainty relation concerns the limitations on the simultaneous determination of physical quantities such as position and momentum; it does not by itself establish the non-existence of an ontically defined position. If an ontically defined position is retained, but its value is experimentally inaccessible with arbitrary precision, the unknown contribution associated with this inaccessibility can be represented by Δx without changing the arithmetic structure of the proof.

Conversely, if a theory rejects the existence of an ontically defined position altogether, it does not constitute a counterexample to the theorem, but instead lies outside the class of theories to which the theorem applies. The distinction between ontic existence and experimental determinability is therefore essential.

B.2 Perturbations of the Coordinate System

A related objection is that the coordinate system itself may be subject to unknown perturbations, such that the numerical values appearing in the measurement equation represent only perturbed or approximate coordinates. Suppose that independent perturbations are associated with the three quantities of the original measurement equation. The resulting relation may be written as

$$\bar{x} + \Delta x_1 = x + \Delta x_2 + x_{\mathrm{ba}} + \Delta x_3. \tag{B.9}$$

All perturbations in Eq. (B.9) enter additively. They can therefore be combined into a single effective contribution,

$$\Delta x = \Delta x_1 + \Delta x_2 + \Delta x_3. \tag{B.10}$$

Equation (B.9) can consequently be rewritten in the form

$$\bar{x} = x + x_{\mathrm{ba}} + \Delta x, \tag{B.11}$$

which is identical in structure to Eq. (B.1).

The three individual perturbations therefore introduce no new mathematical structure into the argument. Since they enter only through their additive combination, their net contribution can be represented by a single effective positional term Δx. The requirement for Δx is analogous to equation (B.2). Thus, the effective contribution Δx yields a finite numerical representation within the extended measurement equation (B.11). Once this effective contribution has been introduced, the analysis of the preceding subsection applies directly. In particular, the three coordinate-conversion procedures remain subject to the conditions

$$\frac{q}{a} \in \mathbb{Z} \setminus \{0\}, \qquad \frac{q}{c} \in \mathbb{Z} \setminus \{0\}, \qquad \frac{q}{e} \in \mathbb{Z} \setminus \{0\}, \tag{B.12}$$

and hence to

$$\frac{q}{ace} \in \mathbb{Z} \setminus \{0\}. \tag{B.13}$$

Thus, perturbations of the coordinate system, or multiple additive perturbations of the spatial description, can be absorbed into a single effective positional contribution. They therefore do not provide a separate mechanism for avoiding the contradiction established by the main theorem.

B.3 Spatially Extended Entities and Field Excitations

A further possibility is to abandon the point-particle description and regard the fundamental entity instead as a spatially extended wave or as an excitation of a field. Such an entity is not pointlike and might therefore initially appear to avoid the no-go result.

The relevant issue, however, is not whether the fundamental entity is pointlike or spatially extended, but whether it possesses an ontically defined spatial property that admits a consistent numerical representation. For example, a uniquely defined spatial feature of a field configuration $\phi(x,t)$, such as a maximum, may be assigned a position

$$x_{\max} = \arg\max_{x} \phi(x,t). \tag{B.14}$$

If $x_{\max}$ is regarded as an ontically defined property of the fundamental entity and is numerically represented in the same position space as the positions considered in the theorem, its determination is subject to the same numerical representation and measurement structure as any other ontically defined position. In particular, the corresponding physical measurement must satisfy the same measurement equation and the same requirements on admissible numerical representations. The fact that the spatial property belongs to an extended field configuration rather than to a pointlike entity does not alter this structure.

Consequently, replacing a point particle by a spatially extended wave or field excitation does not, by itself, provide an escape from the no-go result. An escape would require the fundamental entity to lack any ontically defined spatial property to which the numerical representation assumed by the theorem applies.

More generally, stochastic or deterministic modifications, as well as an increase in the number of spatial dimensions, do not by themselves alter the conclusion. As long as a fundamental entity is assumed to possess an ontically defined and numerically representable position in space, the no-go result applies.

The result therefore does not exclude classical objects or spatially extended physical structures as such. It excludes only those fundamental descriptions in which such an entity is assigned an ontically defined and numerically representable position satisfying the assumptions of the theorem.

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